

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a .

If $(x+2)$ is a factor then $f(-2) = 0$ (3)

$$f(-2) = 2(-2)^3 - 5(-2)^2 + a(-2) + a = 0$$

$$-16 - 20 - 2a + a = 0$$

$$\therefore a = -36$$

4.

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

If $(x-3)$ is a factor then $f(3) = 0$ (2)

$$4(3)^3 - 12(3)^2 + 2(3) - 6 = 108 - 108 + 6 - 6 = 0$$

$\therefore (x-3)$ is a factor of $f(x)$.

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

Comparing coefficients:

$$f(x) = (x-3)(4x^2 + bx + 2) = 4x^3 - 12x^2 + 2x - 6 \quad (4)$$

$$bx^2 - 12x^2 = -12x^2$$

$$\therefore b - 12 = -12$$

$$b = 0$$

$$\therefore f(x) = (x-3)(4x^2 + 2)$$

$$a = 4$$

$$b = 0$$

$$c = 2$$

$$b^2 - 4ac = 0^2 - 4(4)(2) = -32 < 0$$

$\therefore 4x^2 + 2$ has no real roots

hence $x = 3$ is the only real root for $f(x)$.

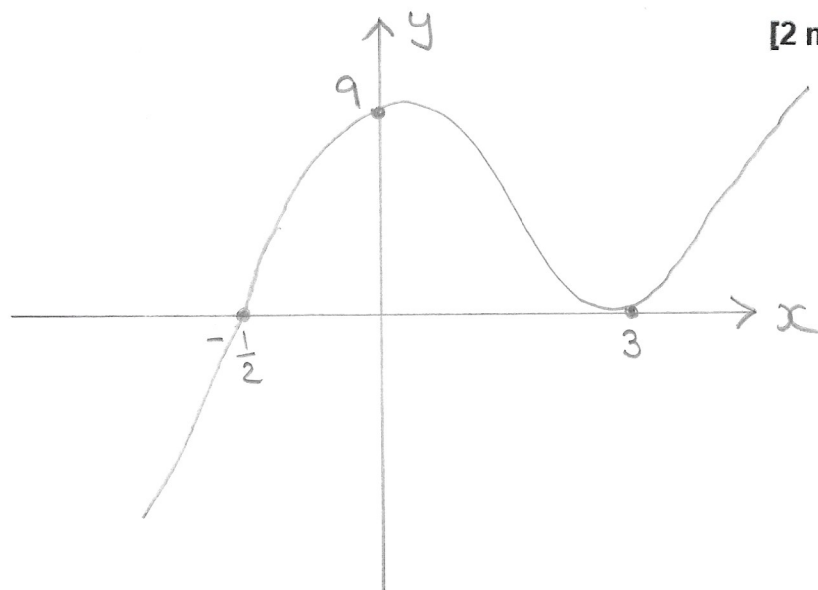
2

Sketch the graph of the curve with equation $y = (2x+1)(x-3)^2$ root at $x = -\frac{1}{2}$ repeated
root at $x = 3$

shape N

y-intercept =

$$1 \times -3 \times -3 = 9$$



[2 marks]

- 8 (a) Find the first three terms, in ascending powers of x , of the expansion of $(1-2x)^{10}$

[3 marks]

$${}^{10}C_0 (-2x)^0 (1)^{10} + {}^{10}C_1 (-2x)^1 (1)^9 + {}^{10}C_2 (-2x)^2 (1)^8 + \dots$$

$$= 1 - 20x + 180x^2 + \dots$$

- 8 (b) Carly has lost her calculator. She uses the first three terms, in ascending powers of x , of the expansion of $(1-2x)^{10}$ to evaluate 0.998^{10}

Find Carly's value for 0.998^{10} and show that it is correct to **five** decimal places.

[3 marks]

$$(1-2x)^{10} = 0.998^{10}$$

$$1-2x = 0.998$$

$$0.002 = 2x$$

$$\therefore x = 0.001$$

$$1 - 20(0.001) + 180(0.001)^2 = 0.98018$$

$$0.998^{10} = 0.9801790434$$

$$= 0.98018 \text{ (5dp)}$$

This matches Carly's value